INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: August 25, 2004)

Name: ___________________________

1. The player throws two dice. What is the probability of getting the sum 6?

2. An urn contains 4 yellow and 16 blue balls. What is the probability of drawing a blue ball from the bag?

3. A fair coin is continually flipped. What is the probability that that first four flips are

   a) H,T,T,H
   b) H,T,H,T

4. Two cards are randomly selected from a deck of 52 playing cards. What is the probability they form a pair?

5. You are playing a game in which another person is rolling a die. You cannot see the die, but you are given information about the outcomes. Your job is to predict the outcome of each roll. Determine the probability that the outcome is a 3, given that you are told the roll has turned up an odd number.

6. An individual who has automobile insurance from a certain company is randomly selected. Let Y be the number of moving violations for which the individual was cited during the last 3 years. The probability mass function of Y is

y	0	1	2	3	4	5
P(y)	.30	.29	.20	.10	.06	.05

What is the expected value of the number of moving violations for which the individual was cited during the last 3 years?

7. In an assembly line, a product must wait before transferring to the following line. If the waiting time (in minutes) has a uniform distribution with a = 0 and b = 20, then what is the expected value for the total waiting time of a product?

8. A box contains 15 percent of defective bulbs. We are looking for a defective bulb, and draw bulbs one by one in random order. What is the probability of making at most four draws to obtain one that is defective? Assume that the percentage of bulbs remains the same after a bulb is drawn.

9. In a facility two assembly lines produce the same product. The rates of defective product in line 1 and 2 are .04 and .10 respectively. At the end of the day one defective product is chosen at random. What is the probability of this product being produced in line 1? Assume that the capacity of product in line 1 and 2 are equal. (Use Baye's Theorem)

10. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3.9 cm and standard deviation 0.3 cm. The second machine produces corks with diameters that are normally distributed with mean 3.99 cm and standard deviation 0.16 cm. Acceptable corks have diameters between 3.85 cm and 4.1 cm. The second machine produces twice as much as the first machine a day. At the end of the day, one acceptable cork is selected at random. What is the probability of that cork to be produced by the first machine?

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: September 15, 2003)

Name: ______________________

1. We have $1400 to invest. All the money must be placed in one of three investments: gold, stock, or money market certificates. If $1400 is placed in an investment, the value of the investment one year from now depends on the state of the economy (see Table below). Assume that each state of the economy is equally likely. For each of the following decision criteria, determine the optimal decision:

   a. maximin
   b. maximax
   c. minimax regret
   d. expected value

Value of $1400	State 1	State 2	State 3
Money Market	$1450	$1450	$1450
Stock	$1500	$1600	$1700
Gold	$1900	$2200	$1000

2. Sodaco is considering producing a new product: Chocovan soda. Sodaco estimates that the annual demand for Chocovan, **D** (in thousands of cases), has the following mass function: P(**D**=30) = .25, P(**D**=50) = .35, P(**D**=80) = .40. Each case of Chocovan sells for $4 and incurs a variable cost of $1.75. It costs $810,000 to build a plant to produce Chocovan. Assume that if $1 is received every year (forever), this is equivalent to receiving $12 at the present time. Considering the reward for each action and state of the world to be in terms of net present value, use each decision criterion (maximin, maximax, minimax regret, and expected value) to determine whether Sodaco should build the plant.

3. Suppose that Pizza King and Noble Greek stop advertising but must determine the price they will charge for each pizza sold. Pizza King believes that Noble Greek's price is a random variable **D** having the following mass function: P(**D** = \$8) = .25, P(**D** = \$11) = .35, P(**D** = \$13) = .4. If Pizza King charges a price p_1 and Noble Greek charges a price p_2, Pizza King will sell 110 + 15(p_2-p_1) pizzas. It costs Pizza King \$3.5 to make a pizza. Pizza King is considering charging \$5, \$6, \$8, \$9, or \$11 for a pizza. Use each decision criterion (maximin, maximax, minimax regret, expected value) to determine the price that Pizza King should charge.

4. Oilco must determine whether or not to drill for oil in the South China Sea. It costs $150,000 and if oil is found the value is estimated to be $720,000. At present, Oilco believes there is a 36% chance that the field contains oil. Before drilling, Oilco can hire (for $14,000) a geologist to obtain more information about the likelihood that the field will contain oil. There is a 52% chance that the geologist will issue a favorable report. Given a favorable report, there is a 71% chance that the field contains oil. Given an unfavorable report, there is a 14% chance that the field contains oil. Determine Oilco's optimal course of action. Also determine EVSI and EVPI.

5. A customer has approached a bank for a $60,000 one-year loan at 12% interest. If the bank does not approve the loan, the $60,000 will be invested in bonds that earn a 6% annual return. Without further information, the bank feels that there is a 6% chance that the customer will totally default on the loan. If the customer totally defaults, the bank loses $60,000. At a cost of $600, the bank can thoroughly investigate the customer's credit record and supply a favorable or unfavorable recommendation. Past experience indicates that

   p(favorable recommendation | customer does not default) = .75
   p(favorable recommendation | customer defaults) = .20

   How can the bank maximize its expected profits? Also find EVSI and EVPI.

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: September 24, 2003)

Name: ______________________

1. The decision sciences department is trying to determine which of two copying machines to purchase. Both machines will satisfy the department's needs for the next ten years. Machine 1 costs $2100 and has a maintenance agreement, which, for an annual fee of $200, covers all repairs. Machine 2 costs $3500, and its annual maintenance cost is a random variable. At present, the decision sciences department believes there is a 40% chance that the annual maintenance cost for machine 2 will be $0, a 30% chance it will be $150, and a 30% chance it will be $250. Assume that if $1 is received every year (forever), this is equivalent to receiving $10 at the present time.

   Before the purchase decision is made, the department can have a trained repairer evaluate the quality of machine 2. If the repairer believes that machine 2 is satisfactory, there is a 60% chance that its annual maintenance cost will be $0 and a 40% chance that it will be $150. If the repairer believes that machine 2 is unsatisfactory, there is a 25% chance that the annual maintenance cost will be $0, a 35% chance it will be $150, and a 40% chance it will be $250. If there is a 60% chance that the repairer will give a satisfactory report, what is EVSI? If the repairer charges $60, what should the decision sciences department do? What is EVPI?

2. The Nitro Fertilizer Company is developing a new fertilizer. If Nitro markets the product and it is successful, the company will earn a $50,000 profit; if it is unsuccessful, the company will lose $41,000. In the past, similar products have been successful 60% of the time. At a cost of $4000, the effectiveness of the new fertilizer can be tested. If the test result is favorable, there is a 80% chance that the fertilizer will be successful. If the test result is unfavorable, there is only a 15% chance that the fertilizer will be successful. There is a 60% chance of a favorable test result and a 40% chance of an unfavorable test result. Determine Nitro's optimal strategy. Also find EVSI and EVPI.

3. Pete is considering placing a bet on the NCAA playoff game between Indiana and Purdue. Without any further information, he believes that each team has an equal chance to win. If he wins the bet, he will win $17,000; if he loses, he will lose $14,000. Before betting, he may pay Bobby $1,500 for his inside prediction on the game; 70% of the time, Bobby will predict that Indiana will win and 30% of the time, Bobby will predict that Purdue will win. When Bobby says that IU will win, IU has a 70% chance of winning, and when Bobby says that Purdue will win, IU has only a 20% chance of winning. Determine how Pete can maximize his total expected profit. What is EVSI? What is EVPI?

4. I am a contestant on the TV show *Remote Jeopardy*, which works as follows. I am first asked a question about Stupid Videos. If I answer correctly, I earn \$200. I believe that I have a 70% chance of answering such a question correctly. If I answer incorrectly, the game is over, and I win nothing. If I answer correctly, I may leave with \$200 or go on and answer a question about Stupid TV Shows. If I answer this question correctly, I earn another \$500, but if I answer incorrectly, I lose all previous earnings and am sent home. My chance of answering this question correctly is .60. If I answer the Stupid TV Shows question correctly, I may leave with my "earnings" or go on and answer a question about Statistics. If I answer this question correctly, I earn another \$800, but if I answer it incorrectly, I lose all previous earnings and am sent home. My chance of answering this question correctly is .40. Draw a decision tree that can be used to maximize my expected earnings. What are my expected earnings?

5. Farmer Jones must determine whether to plant corn or wheat. If he plants corn and the weather is warm, he earns $5500; if he plants corn and the weather is cold, he earns $3500. If he plants wheat and the weather is warm, he earns $4500; if he plants wheat and the weather is cold, he earns $4000. In the past, 35% of all years have been cold and 65% have been warm. Before planting, Jones can pay $500 for an expert weather forecast. If the year is actually cold, there is an 80% chance that the forecaster will predict a cold year. If the year is actually warm, there is an 80% chance that the forecaster will predict a warm year. How can Jones maximize his expected profits? Also find EVSI and EVPI.

6. The NBS television network earns an average of $600,000 from a hit show and loses an average of $200,000 on a flop. Of all shows reviewed by the network, 35% turn out to be hits and 65% turn out to be flops. For $75,000, a market research firm will have an audience view a pilot of a prospective show and give its view about whether the show will be a hit or a flop. If a show is actually going to be a hit, there is a 90% chance that the market research firm will predict the show to be a hit. If the show is actually going to be a flop, there is an 80% chance that the market research firm will predict the show to be a flop. Determine how the network can maximize its expected profits. Also find EVSI and EVPI.

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: October 4, 2004)

Name: ______________________

NOTE THE CHANGES MADE ON PROBLEMS 2-6.

1. In Smalltown, 75% of all sunny days are followed by sunny days, and 25% of all cloudy days are followed by sunny days. Use this information to model Smalltown's weather as a Markov chain.

2. Consider an inventory system in which the sequence of events during each period is as follows: (1) We observe the inventory level (call it *i*) at the beginning of the period. (2) If *i* <= 1, 4- *i* units are ordered. If *i* >= 2, 0 units are ordered. Delivery of units is immediate. (3) With probability .25, 0 units are demanded during the period; with probability .25, 1 unit is demanded during the period; with probability .50, 2 units are demanded during the period. (4) We observe the inventory level at the beginning of the next period.

   Define a period's state to be the period's beginning inventory level. Determine the transition matrix that could be used to model this inventory system as a Markov Chain.

3. A company has two machines. During any day, each machine that is working at the beginning of the day has a .30 chance of breaking down. If a machine breaks down during the day, it is sent to a repair facility and will be working three days after it breaks down (Thus, if a machine breaks down during day 3, it will be working at the beginning of day 6.) Letting the state of the number of machines working at the beginning of the day, formulate a transition probability matrix for this situation.

4. During a given year, 15% of urban families move to the suburbs, and 5% move to a rural location: also, 6% of all suburban families move to an urban location, and 4% move to a rural location; finally, 4% of rural families move to an urban location, and 6% move to a suburban location.

a) If a family now lives in an urban location, what is the probability that it will live in an urban area two years from now? A suburban area? A rural area?

b) Suppose that at present, 40% of all families live in an urban area, 25% in a suburban area, and 35% in an urban area. Two years from now, what percentage of American families will live in an urban area?

5. The following questions refer to Example 1.

a) After playing the game twice, what is the probability that I will have \$3? How about \$2?

b) After playing the game three times, what is the probability that I will have \$4?

6. In Example 2, determine the following n-step transition probabilities:

a) After two balls are painted, what is the probability that the state is [1 0 1]?

b) After three balls are painted, what is the probability that the state is [2 0 0]?

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: October 22, 2003)

Name: ______________________

1. Answer the following questions about the Markov chain below.

P =

	0	1	2	3
0	0.00	0.40	0.00	0.60
1	0.00	0.25	0.00	0.75
2	0.50	0.50	0.00	0.00
3	0.40	0.00	0.60	0.00

a) Are there any transient states?

b) Are there any recurrent states?

c) Do all states communicate?

d) Which states are periodic?

e) Is this chain ergodic?

2. Is this Markov chain ergodic? Explain your answer.

P =

	0	1	2	3
0	0.35	0.00	0.65	0.00
1	0.00	0.20	0.00	0.80
2	0.40	0.00	0.60	0.00
3	0.00	0.30	0.00	0.70

3. Given the following Markov Chain …

		1	2
P =	0	0.40	0.60
	1	0.75	0.25

a) Determine the long run probabilities for each state.
b) Find the mean first passage times for all possible transitions

4. Let's assume that the result of Auburn football games can be described in terms of a Markov chain. Auburn wins 80% of the time following a win, but only 65% of the time following a loss.

    a) Develop the Markov chain for this problem

    b) What is Auburn's expected win-loss record this season (12 games).

    c) If Auburn loses a game, how many games should we expect to suffer through before they win again?

5. Consider two stocks. Stock 1 always sells for $10 or $20. If stock 1 is selling for $10 today, there is a 85% chance that it will sell for $10 tomorrow. If it is selling for $20 today, there is an 75% chance that it will sell for $25 tomorrow. Stock 2 always sells for $15 or $20. If stock 2 sells for $15 today, there is a 80% chance it will sell for $15 tomorrow. If stock 2 is selling for $20 today, there is a 60% chance it will sell for $20 tomorrow. On the average, which stock will sell for a higher price? Also, calculate the mean first passage times for all transitions and explain what they mean.

6. You are playing a game which requires two fair coins. For each turn, you randomly select one of the two coins and flip it up in the air. You then observe the number of heads showing. Below is a list of 3 possible turns.

- Begin with H, T
  i. Randomly select coin 1 … flip … lands on heads … result is H, T
  ii. Randomly select coin 1 … flip … lands on tails … result is T, T
  iii. Randomly select coin 2 … flip … lands on heads … result is T, H

a) Develop the Markov chain for this game with the state being how many heads are showing.

b) Determine the long run probability of 0 heads showing, 1 head showing, 2 heads showing.

c) On average, how many turns does it take to get from 2 heads showing, to 1 head showing.

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: Nov 1, 2004)

Name: ______________________

Solve problem 2 (attached) using Excel, and check your answer by solving the problem by hand. Then make the changes to the problem data as indicated below. Make the changes accumulative one at a time, i.e., make change 2 first, keep it, and then make change 3, and so on. Write your results on this sheet and e-mail your Excel file (name it yourlastname.xls) to lighttk@auburn.edu. The subject of the email should be “yourlastname hw6”. We will test your Excel worksheet with just one of the data sets.

1.

Max Profit =

Best investment:

2.

Project 1				
	\$2	$P(I_1 = 4) = $ **.5**	$P(I_1 = 6) = $ **.4**	$P(I_1 = 8) = $ **.1**

Max Profit =

Best investment:

3.

Project 1				
	\$3	$P(I_1 = 6) = $ **.3**	$P(I_1 = 7) = $ **.4**	$P(I_1 = 10) = $ **.3**

Max Profit =

Best investment:

4.

Project 2	\$1	$P(I_2 = 1) =$ **.6**	$P(I_2 = 2) =$ **.1**	$P(I_2 = 4) =$**.3**

Max Profit =

Best investment:

5.

Project 2				
	\$4	$P(I_2 = 3) =$ **.3**	$P(I_2 = 8) =$ **.4**	$P(I_2 = 9)$ **= .3**

Max Profit =

Best investment:

6.

Project 3				
	\$3	$P(I_3 = 5) =$ **.2**	$P(I_3 = 7)$ **= .4**	$P(I_3 = 8) =$ **.4**

Max Profit =

Best investment:

Problems

Group A

1 In Example 1, find another allocation of milk that maximizes expected daily revenue.

2 Suppose that \$4 million is available for investment in three projects. The probability distribution of the net present value earned from each project depends on how much is invested in each project. Let $\mathbf{I}_t$ be the random variable denoting the net present value earned by project t. The distribution of $\mathbf{I}_t$ depends on the amount of money invested in project t, as shown in Table 2 (a zero investment in a project always earns a zero NPV). Use dynamic programming to determine an investment allocation that maximizes the expected NPV obtained from the three investments.

TABLE 2
Investment Probability for Problem 2

	Investment (millions)	Probability		
Project 1	\$1	$P(\mathbf{I}_1 = 2) = .6$	$P(\mathbf{I}_1 = 4) = .3$	$P(\mathbf{I}_1 = 5) = .1$
	\$2	$P(\mathbf{I}_1 = 4) = .5$	$P(\mathbf{I}_1 = 6) = .3$	$P(\mathbf{I}_1 = 8) = .2$
	\$3	$P(\mathbf{I}_1 = 6) = .4$	$P(\mathbf{I}_1 = 7) = .5$	$P(\mathbf{I}_1 = 10) = .1$
	\$4	$P(\mathbf{I}_1 = 7) = .2$	$P(\mathbf{I}_1 = 9) = .4$	$P(\mathbf{I}_1 = 10) = .4$
Project 2	\$1	$P(\mathbf{I}_2 = 1) = .5$	$P(\mathbf{I}_2 = 2) = .4$	$P(\mathbf{I}_2 = 4) = .1$
	\$2	$P(\mathbf{I}_2 = 3) = .4$	$P(\mathbf{I}_2 = 5) = .4$	$P(\mathbf{I}_2 = 6) = .2$
	\$3	$P(\mathbf{I}_2 = 4) = .3$	$P(\mathbf{I}_2 = 6) = .3$	$P(\mathbf{I}_2 = 8) = .4$
	\$4	$P(\mathbf{I}_2 = 3) = .4$	$P(\mathbf{I}_2 = 8) = .3$	$P(\mathbf{I}_2 = 9) = .3$
Project 3	\$1	$P(\mathbf{I}_3 = 0) = .2$	$P(\mathbf{I}_3 = 4) = .6$	$P(\mathbf{I}_3 = 5) = .2$
	\$2	$P(\mathbf{I}_3 = 4) = .4$	$P(\mathbf{I}_3 = 6) = .4$	$P(\mathbf{I}_3 = 7) = .2$
	\$3	$P(\mathbf{I}_3 = 5) = .3$	$P(\mathbf{I}_3 = 7) = .4$	$P(\mathbf{I}_3 = 8) = .3$
	\$4	$P(\mathbf{I}_3 = 6) = .1$	$P(\mathbf{I}_3 = 8) = .5$	$P(\mathbf{I}_3 = 9) = .4$

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: November 21, 2003)

Name: ______________________

(Problems taken from "Probability Models" by Sheldom Ross)

1. The lifetime of a radio is exponentially distributed with a mean of 8 years. If Jones buys a 8-year-old radio, what is the probability that it will be working after an additional 5 years?

2. Norb and Nat enter a barbershop simultaneously – Norb to get a shave and Nat a haircut. If the amount of time it takes to receive a haircut (shave) is exponentially distributed with mean 20 (15) minutes, and if Norb and Nat are immediately served, what is the probability that Norb finishes before Nat?

3. Cars cross a certain point in the highway in accordance with a Poisson process with the rate of 5 per minute. If Reb blindly runs across the highway, then what is the probability that she will be uninjured if the amount of time that it takes her to cross the road is 6 seconds? (Assume that if she is on the highway when a car passes by, then she will be injured.)

4. Customers arrive at a bank at a Poisson rate λ. Suppose two customers arrived during the first hour. What is the probability that both arrived during the first 15 minutes?

5. Customers arrive at the drive-thru of McDonald's with a Poisson rate of 12 cars per hour. Service times are exponentially distributed with a mean of 3 minutes. You must perform a Monte Carlo Simulation of this process. Download the spreadsheet entitled "MDSim.xls" from WebCT. Use the last four digits of your SSN as a random number seed. Enter this number in the appropriate cell in the spreadsheet, then fill in the data of the missing columns and calculate the average customer waiting time, percent idle time of the Drive-Thru attendant, and average time in system for the customer. (Everything you need to fill in is highlighted in orange). E-mail this portion of the assignment to balcihh@auburn.edu . Do not turn in a printout of this problem.

   NOTE: SINCE EVERYBODY IS ASSIGNED A DIFFERENT SEED, EVERYBODY WILL GET DIFFERENT ANSWERS!!!

INSY 3400 STOCHASTIC OPERATIONS RESEARCH (due date: December 10, 2003)

1. My home uses two light bulbs. On average, a light bulb lasts for 15 days (exponentially distributed). When a light bulb burns out, it takes me an average of 3 days (exponentially distributed) before I replace the bulb.
    a. Formulate a three-state birth-death model of this situation.
    b. Determine the fraction of the time that both light bulbs are working.
    c. Determine the fraction of the time that no light bulbs are working.

2. Problem 1 page 1133.

3. For an $M/M/1/GD/\infty/\infty$ queuing system, suppose that λ is tripled and μ is doubled.
    a. How is L changed?
    b. How is W changed?
    c. How is the steady-state probability distribution changed?

4. A fast-food restaurant has one drive-in window. An average of 24 customers per hour arrives at the window. It takes an average of 3 minutes to serve a customer. Assume that inter-arrival and service times are exponential.
    a. On the average, how many customers are waiting in line?
    b. On the average, how long does a customer spend at the restaurant (from time of arrival to time service is completed)?
    c. What fraction of the time are more than 3 cars waiting for service (this includes the car (if any) at the window)?

5. An average of 45 cars per hour (inter-arrival times are exponentially distributed) are tempted to use the drive-in window at the Hot Dog King Restaurant. If a total of more than 5 cars are in line (including the car at the window) a car will not enter the line. It takes an average of 3 minutes (exponentially distributed) to serve a car.
    a. What is the average number of cars waiting for the drive-in window. On the average, how long will it be before I have received my food?
    b. On the average, how many cars will be served per hour?
    c. I have just joined the line at the drive-in window. On the average, how long will it be before I have received my food?

6. Problem 5 page 1137.

7. An automotive manufacturer is attempting to determine how many dash board assemblers to hire for each shift. During each hour, 55 cars arrive to have a dash board unit installed. Each assembler can handle an average of 30 cars per hour. Assemblers are paid $13/hr and there is a delay cost of $45/hour associated with work in process (i.e. cars that are waiting in line to have dash boards installed). Assuming that an $M/M/s/GD/\infty/\infty$ model is applicable, determine the number of assemblers the company should hire in order to minimize the sum of labor and delay costs.

8. Problem 12 page 1145.