



3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about \$25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?

## 1.6 Something to Chew On – Teacher Notes

### *A Solidify Understanding Task*

**Purpose:**

This task introduces a decreasing arithmetic sequence to further solidify the idea that arithmetic sequences have a constant difference between consecutive terms. Again, connections should be made among all representations: table, graph, recursive and explicit formulas. The emphasis should be on comparing increasing and decreasing arithmetic sequences through the various representations.

**Core Standards:**

**F-BF:** Build a function that models a relationship between two quantities.

**1:** Write a function that describes a relationship between two quantities.\*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

**F-LE:** Linear, Quadratic, and Exponential Models\* (Secondary I focus in linear and exponential only)

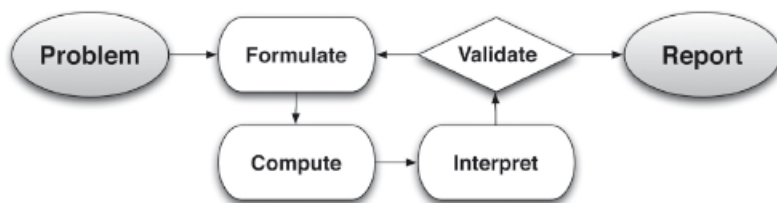
Construct and compare linear, quadratic and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

This task also follows the structure suggested in the Modeling standard:



### Standards for Mathematical Practice of Focus in the Task:

**SMP4 - Model with mathematics.**

**SMP2 - Reason abstractly and quantitatively.**

### The Teaching Cycle:

**Launch (Whole class):** Before handing out the task, ask students to define an arithmetic sequence. Later we will say that it is a linear function with the domain of positive integers. Right now, expect students to identify the constant rate of change or constant difference between consecutive terms. Ask students to give a few examples of arithmetic sequences. Since the only sequences they have seen up to this point have been increasing, expect them to add a number to get to the next term. Then, wonder out loud whether or not it would be an arithmetic sequence if a number is subtracted to get the next term. Don't answer the question or solicit responses.

Read the opening paragraph of the task and be sure that all students understand how the candy machines work; when a quarter is inserted, 7 candies come out. Read the first prompt in the task and discuss what it means to "Represent the number of candies in the machine for a given number of customers." Explain that their representations should include tables, graphs, and equations. As soon as students understand the task, set them to work.

**Explore (Small groups):** To begin the task, students will need to decide how to set up their tables or graphs. Tables should be set with customers as the independent variable and the number of candies as the dependent variable. One way to represent candies versus customers is to calculate how many candies in the machine when it is full and then build tables and graphs by subtracting 7 for each customer. It would be appropriate to have graphing calculators available for this task.

The second prompt (#2) is similar to other increasing geometric sequences that students have previously modeled in Scott’s Workout and Growing Dots. Again, encourage as many representations as possible. When you find that most students are finished with number 1 and 2 it probably a good time to start the discussion. Number 3 is an extension provided for differentiation, but not the focus of the task for most students.

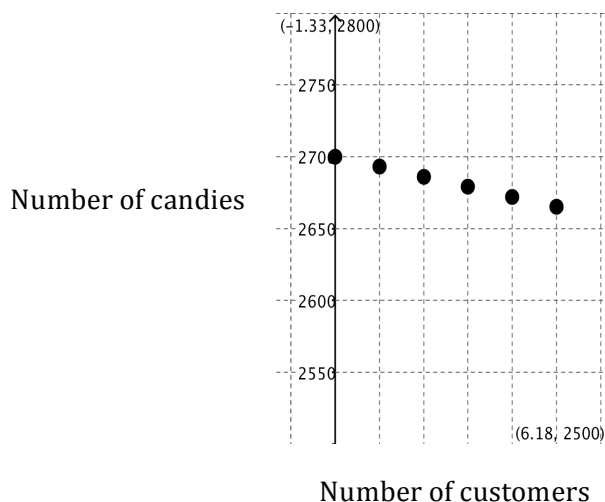
Monitor the group work with particular focus on the work in #1. Have one group prepared to present the table and one group present the graph. Another group can present both forms of the equations from #1. You may choose to have the presenters begin to draw their tables and graphs while the other groups finish their work. Also select just one group to do all of problem #2 on the board for comparison.

**Discuss (Whole Group):** Start the discussion by repeating the question that was stated in the launch: Can you form an arithmetic sequence by subtracting a number from each term to get the next term? Ask the group to present the table that they made for #1, which should look something like this, although students may not have included the first difference at the side. If not, add it in as part of the discussion.

# of Customers	# of Candies	Difference
0	2700	
1	2693	> -7
2	2686	> -7
3	2679	> -7
4	2672	> -7
5	2665	> -7
6	2658	> -7
7	2651	> -7
8	2644	
9	2637	
10	2632	
....	....	
$n$	$2700 - 7n$	

Ask students what they notice about the table. Does the table show a constant difference between terms? What is the constant difference? Does this table represent an arithmetic sequence?

Next, discuss the graph, which should be properly labeled and look something like this:



The constant difference between terms has been demonstrated in the table. Now ask students how this graph of an arithmetic sequence is like the other arithmetic sequences that we have studied in Scott's Workout and Growing Dots. They should identify that the points form a line and the graph is not continuous. (There is no need to emphasize the discrete nature of the sequence since it is a focus in the next module.) How is this graph different? It is decreasing at a constant rate, rather than increasing at a constant rate.

Ask students to compare the recursive formulas for both #1 and #2. Encourage them to use function notation only, so that their formulas look like:

$$1. f(0) = 2700, f(n) = f(n - 1) - 7 \quad 2. f(1) = .25, f(n) = f(n - 1) + .25$$

Ask students, "Based on the recursive formula, is #2 an arithmetic sequence? Why or why not?" Expect students to answer that +.25 shows that each term is increasing by a constant amount.

Now ask students to compare the explicit formulas for both #1 and #2. In function notation, they should be like:

1.  $f(n) = 2700 - 7n$                       2.  $f(n) = .25n$

Ask students how they can identify an arithmetic sequence from an explicit equation. Re-emphasize the definition of an arithmetic sequence as a sequence that has a common difference between consecutive terms. The common difference can be either positive or negative. You may wish to end the discussion by working with students to complete the chart given in the Intervention Activity.

**Aligned Ready, Set, Go Homework: Sequences 1.6**

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Finding the common difference

**Find the missing terms for each arithmetic sequence and state the common difference.**

1. 5, 11, \_\_\_\_\_, 23, 29, \_\_\_\_\_...

Common Difference = \_\_\_\_\_

3. 8, \_\_\_\_\_, \_\_\_\_\_, 47, 60...

Common Difference = \_\_\_\_\_

5. 5, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 25...

Common Difference = \_\_\_\_\_

2. 7, 3, -1, \_\_\_\_\_, \_\_\_\_\_, -13...

Common Difference = \_\_\_\_\_

4. 0, \_\_\_\_\_, \_\_\_\_\_, 2,  $\frac{8}{3}$  ...

Common Difference = \_\_\_\_\_

6. 3, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, -13 ...

Common Difference = \_\_\_\_\_

**SET**

Topic: Writing the recursive function

**Two consecutive terms in an arithmetic sequence are given. Find the recursive function.**

7. If  $f(3) = 5$  and  $f(4) = 8$  ...

$f(5) = \underline{\hspace{1cm}}$ .  $f(6) = \underline{\hspace{1cm}}$ . Recursive Function: \_\_\_\_\_

8. If  $f(2) = 20$  and  $f(3) = 12$  ...

$f(4) = \underline{\hspace{1cm}}$ .  $f(5) = \underline{\hspace{1cm}}$ . Recursive Function: \_\_\_\_\_

9. If  $f(5) = 3.7$  and  $f(6) = 8.7$  ...

$f(7) = \underline{\hspace{1cm}}$ .  $f(8) = \underline{\hspace{1cm}}$ . Recursive Function: \_\_\_\_\_



**Two consecutive terms in a geometric sequence are given. Find the recursive function.**

10. If  $f(3) = 5$  and  $f(4) = 10$  ...

$f(5) = \underline{\hspace{2cm}}$ .  $f(6) = \underline{\hspace{2cm}}$ . Recursive Function:  $\underline{\hspace{10cm}}$

11. If  $f(2) = 20$  and  $f(3) = 10$  ...

$f(4) = \underline{\hspace{2cm}}$ .  $f(5) = \underline{\hspace{2cm}}$ . Recursive Function:  $\underline{\hspace{10cm}}$

12. If  $f(5) = 20.58$  and  $f(6) = 2.94$  ...

$f(7) = \underline{\hspace{2cm}}$ .  $f(8) = \underline{\hspace{2cm}}$ . Recursive Function:  $\underline{\hspace{10cm}}$

## GO

Topic: Evaluating using function notation

**Find the indicated values of  $f(n)$ .**

13.  $f(n) = 2^n$  Find  $f(5)$  and  $f(0)$ .

14.  $f(n) = 5^n$  Find  $f(4)$  and  $f(1)$ .

15.  $f(n) = (-2)^n$  Find  $f(3)$  and  $f(0)$ .

16.  $f(n) = -2^n$  Find  $f(3)$  and  $f(0)$ .

17. In what way are the problems in #15 and #16 different?

18.  $f(n) = 3 + 4(n - 1)$  Find  $f(5)$  and  $f(0)$ .

19.  $f(n) = 2(n - 1) + 6$  Find  $f(1)$  and  $f(6)$ .